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Letter to the Editor

Reply to Comment by A. J. Chorin

The scientific content of Professor Chorin's comments seems to be based on a misconception, and it is necessary therefore to reiterate the purpose of the calculation by Dr. Milinazzo and myself.

The use of discrete point vortices to calculate approximations to the two-dimensional motion of an inviscid incompressible fluid is a popular and fairly old method. It was used by Rosenhead [1] over 45 years ago and doubtless earlier work exists. The recent discovery [2] that the evolution of some turbulent flows looks like the interaction of finite-cored vortices is currently stimulating several studies. In the last decade, the method of point vortices has been extended to incorporate a small viscosity. This is important as real flows cannot be exactly inviscid (we exclude He II, for which point vortices are a real physical phenomenon), and the dependence on Reynolds number may be of considerable physical interest.

Unfortunately, the accuracy of the method is uncertain, both for inviscid flows and for the viscous extensions. The main evidence is that the flows "look right," i.e., the flow patterns are in accordance with intuitive ideas, or that certain gross features such as mean drag are in agreement with experiment. These properties are relatively insensitive to the details of the flow field and do not provide conclusive support. Moreover, experimental flows in fluids of small viscosity (or more precisely at large Reynolds number) are usually turbulent and hence three dimensional. The ability of the method to model turbulent flow is a rather different question from its ability to represent accurately two-dimensional solutions of the Euler or Navier–Stokes equations.

In accordance with standard scientific procedure, we tested a feature of the method by comparing it with an exact solution. The particular feature to be tested was the idea that viscous diffusion can be represented by adding a random walk component to the displacement of the individual point vortices. For this purpose we used an initial condition for which both the inviscid and viscous solutions are known exactly so that the exact effects of viscosity are known. The error in the calculation of viscous effects by the random walk method was determined by comparing the calculated values of changes due to viscosity with the exact values of the same quantities. Professor Chorin calls this procedure "unusual" and implies that a large error found in this way is of no significance. In my opinion, it is the only sensible way to measure the error.

The particular flow which we examined was the isolated line vortex of finite cross section. This is a basic flow field for many problems of fluid mechanics, and a method which claims to be able to calculate viscous effects at large Reynolds number should be able to calculate accurately the effect of viscosity on a single vortex. Professor

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Chorin implies that since the effect of viscosity on a single vortex is small, it does not matter that it is not calculated accurately and that criticism on this ground is inappropriate. I disagree. One can anticipate that significant error might well occur in problems such as the amalgamation or disintegration of finite-cored vortices, where the phenomenon may be sensitive to the amount of viscous diffusion, if the diffusion of a single vortex is not accurately represented.

I am unable to understand Professor Chorin's statement that our cutoff is obviously equivalent to no cutoff at all. The calculation "blew up" if there was no cutoff, and we picked a small value with which the *inviscid* flow field was found to be calculated accurately. The nonlinear terms vanish in the exact solution, but not in the discrete point vortex representation. Professor Chorin's cutoff was selected on grounds connected with the formation of new vortices at a rigid wall in order to satisfy the no-slip boundary condition and was proportional to the length of the segments into which the boundary was divided. The relevance of this criterion for the diffusion and motion of vorticity away from walls is obscure and is *not* discussed in [3].

References

1. L. Rosenhead, *Proc. Roy. Soc. A* **134** (1931), 170.
2. G. L. Brown and A. Roshko, *J. Fluid Mech.* **64** (1974), 775.
3. A. J. Chorin, *J. Fluid Mech.* **57** (1973), 785.

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